ELASTODYNAMIC RESPONSE OF THE GROUND SURFACE CAUSED BY WIND

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Abstract

Wind-induced ground motion is one of the main sources of noise in seismic surveys, which can cause difficulties in the data interpretation. However, it might be exploited as a seismic source for investigating near surface soil properties. In a previous study, the ground displacement spectra was predicted based on a quasi-static linear elastic ground model. The model predicted a significant difference between vertical and horizontal displacements due to a vertical pressure whereas; the field measurements indicated similar magnitudes of horizontal and vertical displacement.

In an attempt to better understand the ground displacement due to interactions with the wind, we present an expanded prediction model based on elastodynamic theory for the surface displacements of an elastic half-space due to a harmonic vertical surface load as a function of frequency and apply it in the original wind-ground prediction model. In addition, we use COMSOL-Multiphysics® in order to confirm theoretical predictions for the dynamic response of the ground due to the surface load. The simulation results and theoretical predictions are closely matched with each other.

The new prediction model developed based on the elastodynamic theory is compared with the previous quasi-static model and experimental data. Both the static and dynamic models of the ground predict similar vertical displacements of the ground. Although the dynamic model predicts a slightly larger horizontal displacement than the static model, it is still much smaller than the measured displacements.

Introduction

Wind noise is an unwanted source of seismic vibrations that obscures or masks seismic data. Understanding and predicting this noise source is an important step in reducing its effects. Wind noise on seismic sensors can be attributed to the direct interaction of the wind with the sensor or the ground vibrations generated by the turbulent pressure and shear stress of wind on the ground surface. In other words, wind is a distribution of vertical and possibly horizontal surface loads that shake the ground and obscure the desirable seismic phenomena that we want to observe.

Naderyan et al. (2015) utilized the theory by Yu et al. (2011) to predict wind pressure at the ground surface based on the wind velocity and the frequency-dependent correlation of wind noise founded by Shields (2005) to estimate the distribution of wind pressure on the surface. The theory of elasticity for a homogeneous elastic half-space was used to develop a quasi-static model for wind-induced ground displacements. The predictions, assuming that the wind produces only vertical forces on the ground, were compared with field measurements and indicated a good agreement for the vertical displacement. However, the predicted horizontal displacement was much smaller than the measured values. In order to account for the horizontal deformation, they speculated that the wind must apply a horizontal load (shear stress) with the same order of magnitude as the vertical pressure.

In quasi-static loading, the load is applied over a sufficiently long period and so slowly that the displacement of the model can be considered static. As we are interested in a frequency range above 4.5Hz, we replace the quasi-static theory with the dynamic theory to model the ground surface motions more realistically.

In the following section, we provide a summary of both quasi-static and elastodynamic models for the displacements of an elastic homogeneous half space due to a vertical surface load. Although the radial to vertical displacement ratio of dynamic deformations is very close to the static deformations at points very close to the source, the dynamic ratio increases with range and has a frequency dependent oscillation. In the final section, we present the general wind-ground model which can be used with the static or the dynamic surface response of the ground. Since there is no closed form function for the dynamic response, it is generated by an interpolation of the computed data points and applied to the wind-ground model. Finally, we compare new and previous results specifically the ratio of vertical to horizontal displacement of the ground surface for each case.

Ground Deformation due to Surface Loading

Static Loading

In previous studies, the ground was assumed to be a homogeneous linearly elastic medium bounded by an infinite plane on one side (half-space). The vertical and radial surface deformations of the model due to a normal surface load, as a function of radius from a delta function force source are (Landau and Lifshitz, 1986):

$$u_r(r) = \frac{(1+\nu)(1-2\nu)}{2\pi E}\frac{1}{r}F_z \tag{1a}$$

$$u_z(r) = \frac{(1+\nu)(1-\nu)}{\pi E}\frac{1}{r}F_z \tag{1b}$$

where r is the radial distance from the load's center, ν and E are Poisson's ratio and Young's modulus, respectively, and F_z is the vertical force. As the equations show, both components of the displacements decay with range as a function of $1/r$ which has singularity point at the center.

Dynamic Loading

To our knowledge, there is no closed form solution for the dynamic response of an elastic, isotropic and homogeneous half-space, subjected to a periodic normal load. The most applicable solution found in the literature is for a harmonic pressure force $P_z(r,t)$ distributed uniformly and axial-symmetrically over a circular region of the surface with radius of r_0. The radial and vertical displacements on the surface, excited by the above-described force are expressed by (Sung, 1954):

$$u_r(r) = -\frac{F_z}{\pi G r_0}\mathrm{P}\int_0^\infty \frac{2\xi^2 - k^2 - 2\sqrt{\xi^2-h^2}\sqrt{\xi^2-k^2}}{(2\xi^2-k^2)^2 - 4\sqrt{\xi^2-h^2}\sqrt{\xi^2-k^2}\,\xi^2}\,\xi\, J_1(\xi r_0)J_1(\xi r)\,d\xi - i\frac{F_z}{2G}\xi_0 H J_1(\xi_0 r) \tag{2a}$$

$$u_z(r) = \frac{F_z}{\pi G r_0}\mathrm{P}\int_0^\infty \frac{\sqrt{\xi^2-h^2}\,k^2}{(2\xi^2-k^2)^2 - 4\sqrt{\xi^2-h^2}\sqrt{\xi^2-k^2}\,\xi^2}\,J_1(\xi r_0)J_0(\xi r)\,d\xi - i\frac{F_z}{2G}\xi_0 K J_0(\xi_0 r) \tag{2b}$$

where r is radial distance from the load's center, F_z is the amplitude of oscillating load, P means " the principal value of ", G is shear modulus, J is Bessel function of first kind, ξ_0 is the root of the denominators, H, K, h and k are defined as:

$$H = -\frac{2(2\xi_0^{\,2} - k^2 - 2\sqrt{\xi_0^{\,2} - h^2}\sqrt{\xi_0^{\,2} - k^2})}{d((2\xi^2 - k^2)^2 - 4\sqrt{\xi^2 - h^2}\sqrt{\xi^2 - k^2}\,\xi^2)/d\xi(\xi_0)\,r_0} J_1(\xi_0 r_0) \tag{2c}$$

$$K = -\frac{2\sqrt{\xi_0^{\,2} - h^2}\,k^2}{d((2\xi^2 - k^2)^2 - 4\sqrt{\xi^2 - h^2}\sqrt{\xi^2 - k^2}\,\xi^2)/d\xi(\xi_0)\,\xi_0\,r_0} J_1(\xi_0 r_0) \tag{2d}$$

$$h^2 = \frac{\rho\omega^2}{\lambda + 2G} \tag{2e}$$

$$k^2 = \frac{\rho\omega^2}{G} \tag{2f}$$

where λ is the first Lamé parameter and ρ is density. The vertical and radial displacements can be numerically calculated at a series of points on any radial line from the source, using eq. 2.

A 2-D axisymmetric rectangle subjected to a vertical harmonic surface load, shown in Fig. 1, was generated in COMSOL Multiphysics® and computations were conducted for different frequencies. The problem is solved in both frequency domain and time domain in the COMSOL FEA simulations.

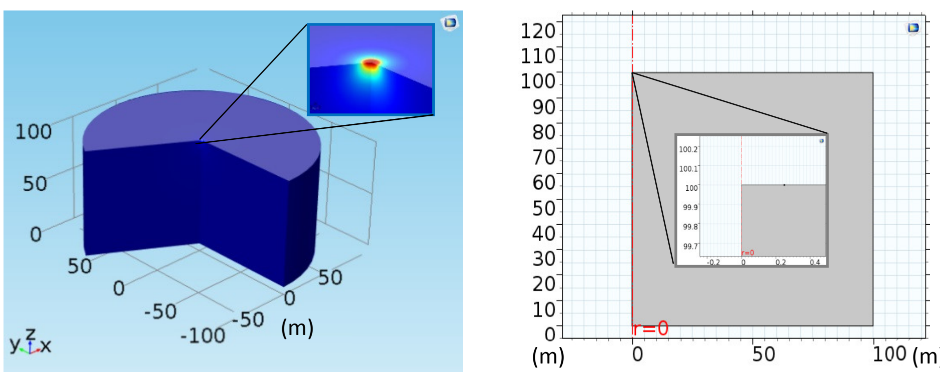

Figure 1: 3-D and 2-D view of simulation geometry and the source location.

The soil and source properties of model are listed in Table 1. The displacement amplitudes as a function of distance from 10Hz and 100Hz sources are shown in Fig. 2. There is close agreement between the theory and the simulation results from the FEA. At short distances from the source the vertical displacements are similar for both frequencies but there is a significant difference for the near offset horizontal displacement. Both vertical and horizontal displacements decrease in a similar fashion as a function of range but the decay is larger for the lower frequency. A series of waves propagate from the source with P-wave, S-wave and surface wave (Rayleigh wave) velocities. The body wave amplitudes decay very rapidly therefore, at a sufficient distance from the source, amplitudes of the other waves are small in comparison with the amplitudes of the surface waves and the deformation ratios are converging to the Rayleigh wave horizontal to vertical deformation ratio. Because of the interference of the waves in the nearfield, the displacement amplitudes do not decay monotonously in proportion to long distance from the source (Barkan, 1960).

Table 1: Simulation Parameters

Force Amplitude (N)	**Load Radius (m)**	**P-wave Velocity (m/s)**	**S-wave Velocity (m/s)**	**Density (kg/m^3)**
50	0.25	300	170	2000

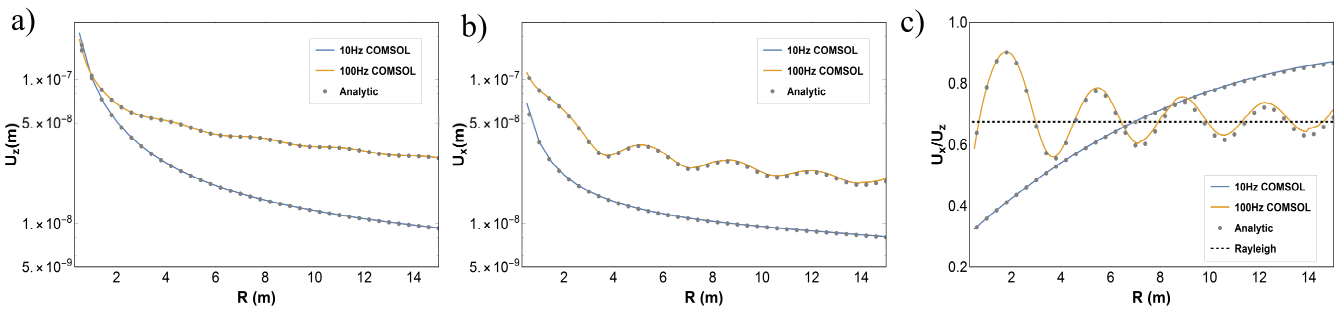

Figure 2: Simulation and analytical results for (a) the vertical displacement, (b) horizontal displacement and (c) horizontal to vertical displacement ratio.

A theoretical comparison between static and dynamic responses of the ground surface $(z = 0)$ due to a surface load is presented in Fig. 3. In the near field, the predicted ground displacements are close for both static and dynamic models. As the distance increases from the source, the dynamic model predicts larger vertical and horizontal deformations. In addition, the horizontal to vertical ratio increases with range and oscillates around the Rayleigh wave deformation ratio. However, the static deformation ratio is only a function of Poisson's ratio and remains constant with range because, both vertical and horizontal deformations, predicted by Eq. 1, are proportional to $1/(r\,E)$. In the dynamic case, the equations includes Bessel functions and decays with range and their ratio converges to the Rayleigh wave ratio, which is a function of elastic parameters of the ground. In reality, the dynamic displacements are supposed to decay faster because of the soil attenuation.

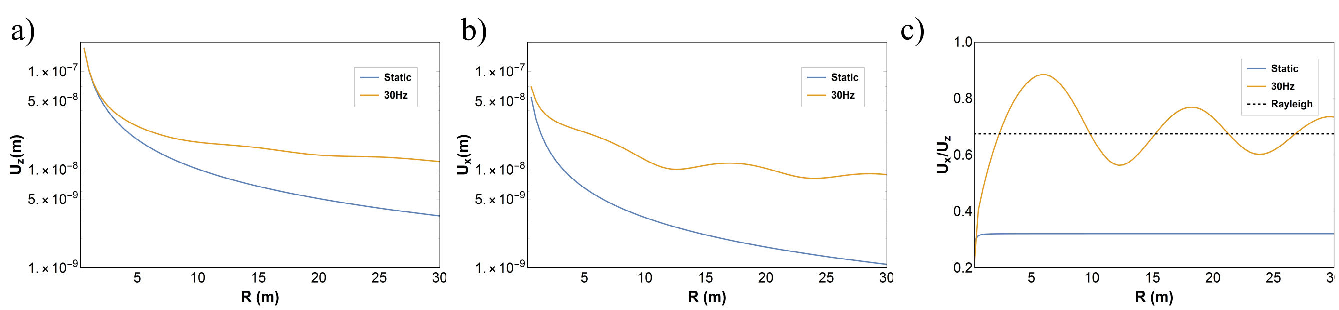

Figure 3: Static and dynamic responses of (a) the vertical displacement, (b) horizontal displacement and (c) horizontal to vertical displacement ratio.

Wind-Ground Coupling Theory

Naderyan et al. developed a wind-ground coupling theory by combining following theories: the ground (half-space) deformations due to surface loads (Landau and Lifshitz, 1986), the wind pressure spectrum at the ground surface from the measured wind velocity profile (Kraichnan, 1956, Raspet et al., 2008, Yu et al., 2011), and the distribution of sources associated with wind turbulence over the ground surface applied by the wave number-dependent correlation function of the wind noise in the downwind and crosswind directions (Shields, 2005). Combining these theories leads to following equations for the power spectra of the vertical and horizontal components of the wind-induced ground surface displacements due to the vertical pressure (Naderyan et al., 2015).

$$|U_r(k)|^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_r(r)u_r(r')p_z^{\,2}(k) \exp(-\alpha\frac{k}{2\pi}|x-x'|)\cos(k|x-x'|)\exp(-\beta\frac{k}{2\pi}|y-y'|)\,dxdx'dydy' \tag{4a}$$

$$|U_z(k)|^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u_z(r)u_z(r')p_z^{\,2}(k) \exp(-\alpha\frac{k}{2\pi}|x-x'|)\cos(k|x-x'|)\exp(-\beta\frac{k}{2\pi}|y-y'|)\,dxdx'dydy' \tag{4b}$$

where U_r and U_z are horizontal and vertical displacements at the observation point of the pressure, k is wind wave number, υ and E are Poisson's ratio and Young's modulus, respectively, (x, y) and (x', y') are coordinates of two random points on the ground surface with respect to the observation point of the pressure, r and r' are $\sqrt{x^2+y^2}$ and $\sqrt{x'^2+y'^2}$ respectively, $p_z^{\,2}$ is the Power Spectral Density (PSD) of wind pressure on the ground, α and β are two constant numbers based on the wind velocity. In the study by Naderyan et al. (2015) the closed form expressions for the ground deformation given by Eq. 1 can be inserted into Eq. 4. Numerical integration (over truncated ranges) is used to calculate the PSD. For the dynamic case, the displacements given by Eq. 3 are in the form of integrals with no closed-form solution to substitute in Eq. 4. Therefore, an interpolation function calculated based on the integral answers at a finite number of points is substituted in the prediction equations (Eq. 4) and the revised version of the integrals are numerically computed. Fig. 4 displays the new, previous (Naderyan et al., 2015) predictions and experiment results versus frequency. The predictions for the vertical component of the displacement match very well. The horizontal displacements of the dynamic model are larger than the previous predictions. The previous study showed the PSD of the vertical displacements due to the normal pressure to be about 17 times greater than the horizontal. However, in this study by applying elastodynamic equations, this ratio reduces to about 11, without changing the vertical component. Although the new method increases the horizontal component, it is still much less than the vertical.

The ground deformation part of Eq. 4 obtained from Eq.1 and Eq.3, is a decaying function of range approaching to infinity at $r = 0$. In addition, the integrals are taken over the range in Eq. 4. These two mathematical facts can reveal the significant effect of the near field where the vertical displacements are significantly larger than the horizontal for both static and dynamic responses (Fig. 3) and explain the small difference between the dynamic and static horizontal components.

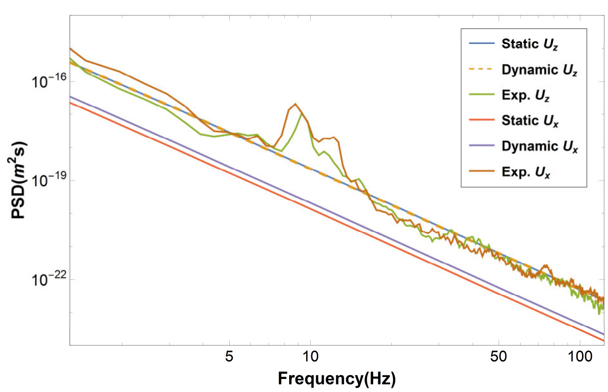

Figure 4: The predictions and the experimental results.

Conclusion

In the previous study, a theoretical model transfers the driving pressure perturbations on the ground surface to the ground vibrations. This paper proposes a new prediction by replacing the static response of a linear elastic half-space (ground) with the dynamic response caused by the wave propagation due to a harmonic surface load. Sung's theoretical solution is used for the ground vibrations and results are computed for a series of observation points on the surface.

A computational simulation model is built in COMSOL Multiphysics® and run in the time and frequency domains to confirm the analytic solution. The results show a perfect agreement between the frequency domain, the time domain, and the theory response which confirms the reliability of the solution.

The new predictions are obtained by assuming a dynamic response for the ground instead of the previously assumed quasi-static response. The results show a good agreement with the previous study for the vertical component of displacements, however, the horizontal component increases, which is due to its oscillating behavior and also larger value of horizontal to vertical displacement ratio with respect to the static theory. Although this dynamic approach decreases the difference between the predicted and measured horizontal displacement, it does not adequately account for the difference. The deformation equations (Eq. 1 & 3) weight the overall results to deformation in the near field rather the far field where the normal displacement is notably larger than the radial.

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