ASSESSING QUANTITATIVELY INTERPRETABLE ZONES FROM 1D FORWARD MODELLING AEM INVERSION MODELS

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Summary

The inversion of time-domain AEM data with full 3D inversion requires specialists' expertise and a huge amount of computational resources, not readily available to everyone. Consequently, quasi-2D/3D inversion methods are prevailing, using a much faster but approximate (1D) forward model. The question remains whether the obtained inversion results are reliable and can be interpreted quantitatively. We propose an appraisal tool for quasi-2D/3D methods that indicate zones in the inversion model that are not in agreement with the multidimensional (2D/3D) forward model and therefore, should not be interpreted in a quantitative fashion.

The image appraisal step only requires one full 2.5D or 3D forward and one multidimensional Jacobian computation on a coarse mesh to compute a so-called normalized gradient. Large values in that gradient indicate model parameters that do not fit the true multidimensionality of the observed data well and should not be interpreted quantitatively. We demonstrate our method on a real AEM survey in a salinization context, revealing problematic zones in the fresh-saltwater interface. Interestingly, the problematic zones are not necessarily at larger depths, but where the interface is changing. The advantage of our approach is that all computations are feasible on a single laptop.

Assessing quantitatively interpretable zones from 1D forward modelling AEM inversion models

Introduction

The Airborne ElectroMagnetic induction (AEM) method is a practical tool to map near-surface geological features over large areas, as electromagnetic induction methods are sensitive to the bulk resistivity. While the AEM systems have massively advanced within the last decade (Auken et al., 2017), the data interpretation process and the related computational burden remains a main impediment. Full 3D inversion requires specialists' expertise and a huge amount of computational resources (Engebretsen et al., 2022; Cox et al., 2010), not readily available to everyone. Consequently, quasi-2D and quasi-3D inversion methods are prevailing, using a much faster but approximate (1D) forward model. While using an 1D forward model is valid for slowly varying lateral variations, it is often adopted for regions where this hypothesis is not valid. The question remains whether the obtained inversion results are reliable and can be interpreted quantitatively, in which we enter the field of AEM image appraisal methods (Alumbaugh and Newman, 2000; Christiansen and Christensen, 2003).

In this work, we propose an appraisal tool for quasi-2D/3D methods that indicate zones in the inversion model that are not in agreement with the multidimensional (2D/3D) forward model and therefore, should not be interpreted in a quantitative fashion. The advantage of the approach is that all computations are feasible on a single laptop.

Method

Forward modelling
The forward model describes the subsurface's response to a specific subsurface realization and a specific survey set-up. There are two main common approaches: The first is based on (semi-)analytical models that solve the (continuous) Maxwell equations for a one dimensional subsurface model, meaning that it assumes horizontal layers without lateral variations. An open-source Python implementation by Werthmüller (2017) neatly implements such a forward model in a fast and reliable fashion. We refer to this model as the low-fidelity model. The second approach is based on a discretization of the physics on a mesh. Those simulations mimic the full 3D soil response of the potentially non-1D subsurface and allow for multidimensional modelling. In this work, the finite volume method from the open-source package SimPEG (Heagy et al., 2017) is used. With a suitable discretization of the geometry, an accurate magnetic field response can be obtained and we refer to these simulations as the high-fidelity model.

Quasi-2D inversion
An inversion model $\mathbf{m}$ fits the observed data $\mathbf{d}^{\text{obs}}$ and is simple in Occam's sense (Constable et al., 1987). This is accomplished by minimizing an objective function

$$\phi(\mathbf{m}) = \phi_d(\mathbf{m}) + \beta\phi_m(\mathbf{m}), \quad (1)$$

where ϕ_d and ϕ_m are, respectively, the data and model misfit. β is a regularization parameter which balances the relative importance of the two misfits.

In quasi-2D inversion, the data misfit

$$\phi_d(\mathbf{m}) = \frac{1}{n} \left|\left|\mathbf{W}_d \left(\mathbf{d}^{\text{obs}} - \mathscr{F}_{\text{1D}}(\mathbf{m})\right)\right|\right|_2^2 \tag{2}$$

uses an 1D approximation for the forward model $\mathscr{F}_{\text{1D}}(\mathbf{m})$, which significantly reduces the computation time of the inversion procedure. In this work, the model misfit ϕ_m promotes smooth solutions (Constable et al., 1987) and a relative regularization parameter α balances the lateral complexity with respect to the vertical complexity.

The optimal regularization parameter β is selected via the chi-squared criterion, meaning that an optimal inversion model fits the observed data to an error weighted RMS, where the uncorrelated noise is defined in the diagonal matrix $\mathbf{W}_d$. As we work with two forward models, we distinguish two RMS errors:

$$\varepsilon_{\text{1D}} = \sqrt{\frac{1}{n} ||\mathbf{W}_d \left(\mathbf{d}^{\text{obs}} - \mathscr{F}_{\text{1D}}(\mathbf{m})\right)||_2^2}, \qquad \varepsilon_{\text{2.5D}} = \sqrt{\frac{1}{n} ||\mathbf{W}_d \left(\mathbf{d}^{\text{obs}} - \mathscr{F}_{\text{2.5D}}(\mathbf{m})\right)||_2^2}, \tag{3}$$

where $\mathscr{F}_{\text{2.5D}}$ refers to the high fidelity model which allows for 2D variations on a 3D mesh.

Sensitivity

The multidimensional sensitivity matrix or Jacobian $\mathbf{J}\left(=\frac{\partial \mathbf{d}}{\partial \mathbf{m}}\right)$ is required to map poorly fitting data points to specific areas of the inversion model. A high sensitivity value signifies that a change of this parameter influences the predicted data strongly whereas a low value denotes less or no influence of this parameter on the predicted data. The sensitivity matrix $\mathbf{J}$ is computed on a coarse 2.5D mesh with a strongly reduced mesh size, allowing for a computation on a single laptop. This is a combination of the moving footprint approach (Cox et al., 2010) and the approach proposed by Zhang et al. (2021).

Since we are interested in the relative importance of each zone in the inversion model, we use the normalized sensitivity matrix $\tilde{\mathbf{J}}$, which normalizes the sensitivity function (each row in $\mathbf{J}$) for each datum. As we wish to identify zones where the 1D approximation might induce a significant error in the inversion model, we combined the normalized sensitivity with the 2.5D data misfit term to compute the normalized gradient of the data misfit with the high-fidelity forward model

$$\tilde{\nabla}\phi_{d,2.5D} = \tilde{J}^{\text{T}}\mathbf{W}_d \left(\mathbf{d}^{\text{obs}} - \mathscr{F}_{\text{2.5D}}(\mathbf{m})\right), \tag{4}$$

which gives an indication on which model parameters would change in a full 2D inversion, meaning that those model parameters do not fit the data well with a multidimensional forward model. Put differently, model parameters that would not initially change are likely to be fitting the data well and can be interpreted quantitatively.

The normalized gradient $\tilde{\nabla}\phi_{d,2.5D}$ provides information whether the problematic model parameters are likely overestimated or underestimated. While this may be helpful, we should not be inclined to interpret these values quantitatively.

Proposed workflow for low-cost image appraisal

1. After each quasi 2D or 3D inversion result, verify the RMS error with a high-fidelity forward model $\varepsilon_{\text{2.5D}}$.
2. If the reliable RMS error is problematic, compute the approximated sensitivity matrix for each data point. Compute the normalized gradient $\tilde{\nabla}\phi_{d,2.5D}$ to determine the problematic zones.
3. Look at how the normalized sensitivity would alter the inversion model. We avoid the trap of interpreting this information quantitatively.
4. If specific features cannot be interpreted quantitatively, crucial to the research objectives, then a full 2D/3D inversion is warranted (if sufficient computational resources are available).

Results

In this section, we apply our proposed methodology to a real field data case within a saltwater intrusion context (Delsmans et al., 2019) , with time-domain AEM data from a SkyTEM instrument. The inversion model in Figure 1A is obtained via quasi-2D inversion. The relative regularization parameter α is set to 10 and the regularization parameter β is determined via the chi-squared criterion. The obtained inversion model has ε_{1D} of 0.97. The result shows low values of Electrical Conductivity (EC) at shallow depths and high values of EC between 20 m and 50 m depth, which corresponds to a saltwater lens resting on a clay layer (via petrophysical laws, the values of EC can be mapped to salinity).

To assess whether the inversion model is reliable, we compare the data from a high-fidelity model $\mathscr{F}_{2.5D}(\mathbf{m})$ with the observed data $\mathbf{d}^{obs}$. The multidimensional predicted data is obtained with simulations in SimPEG. The reliable RMS $\varepsilon_{2.5D}$ is equal to 2.39, which signals that the 1D assumption was invalid for some areas in the inversion model. For example, the data at time channel 26 ($t = 0.1722$ ms) is shown in Figure 1C. It is apparent that the high-fidelity predicted data is mostly not within the error bands. Even at $X = 570$ m, where the 2.5D data is in agreement with the observed data, the underlying sensitivity in Figure 1B reveals that the data point is most sensitive to an anomaly $\pm$ 40 m to the left side, which is impossible to obtain with 1D forward modelling.

As the $\varepsilon_{2.5D}$ is significantly greater than 1, we need to localize the areas in the inversion model that are problematic. Therefore, the normalized gradient $\tilde{\nabla}\phi_{d,2.5D}$ is computed with the approximate sensitivities from the reduced, coarser modelling mesh. The threshold on the normalized gradient depends on whether an optimistic or conservative analysis is made. In our work, we set the threshold for the optimistic image appraisal to 25% of the maximum value of the normalized gradient $\tilde{\nabla}\phi_{d,2.5D}$ and to 10% for the conservative appraisal.

Both the optimistic and conservative appraisal are shown in Figure 2. The optimistic case reveals that specific anomalies below the surface and specific regions of the fresh-saltwater interface cannot be quantitatively interpreted. The conservative appraisal adds an extra anomaly below the surface and dictates that the entire fresh-saltwater interface should not be interpreted quantitatively. Interestingly, the problematic zones are not necessarily at larger depths, but where the interface is changing. In those zones, the 1D approximation is no longer valid and a high-fidelity model should be used. The main body of freshwater right below the surface and saltwater lens, however, can be quantitatively interpreted via quasi-2D inversion.

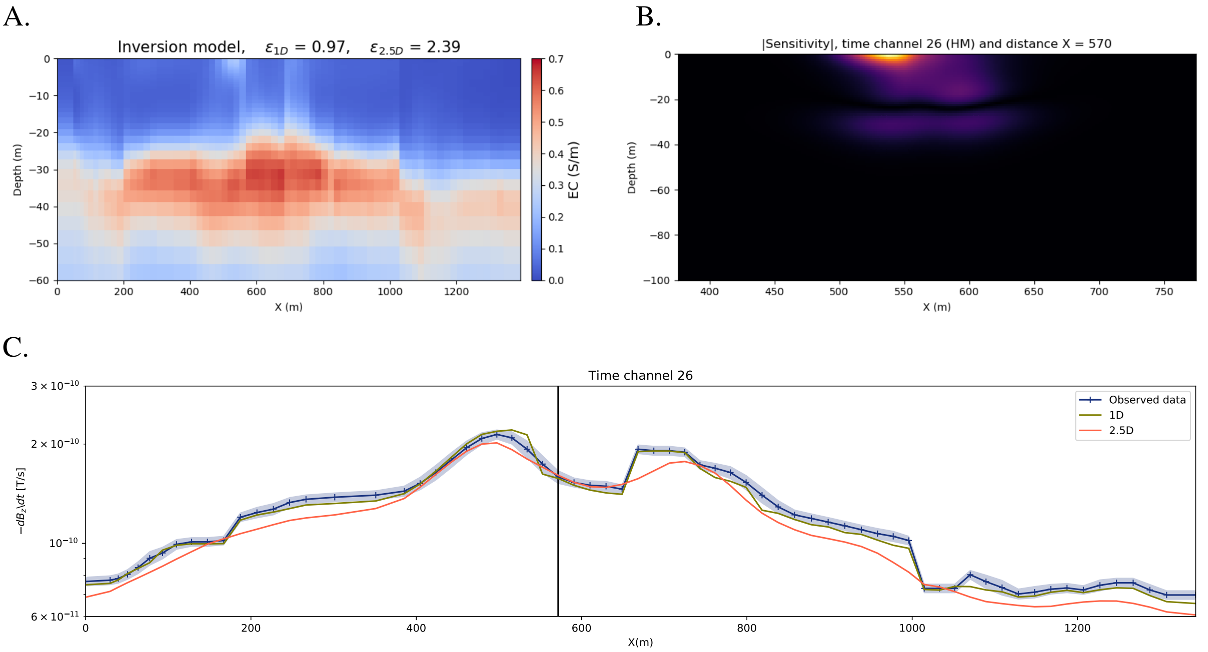

Figure 1 *A. Inversion model obtained with quasi-2D inversion. B. The sensitivity to a data point at 570m and 0.1722 ms. C. Comparison between the observed data* $\mathbf{d}^{obs}$*,* $\mathscr{F}_{1D}(\mathbf{m})$ *and* $\mathscr{F}_{2.5D}(\mathbf{m})$*.*

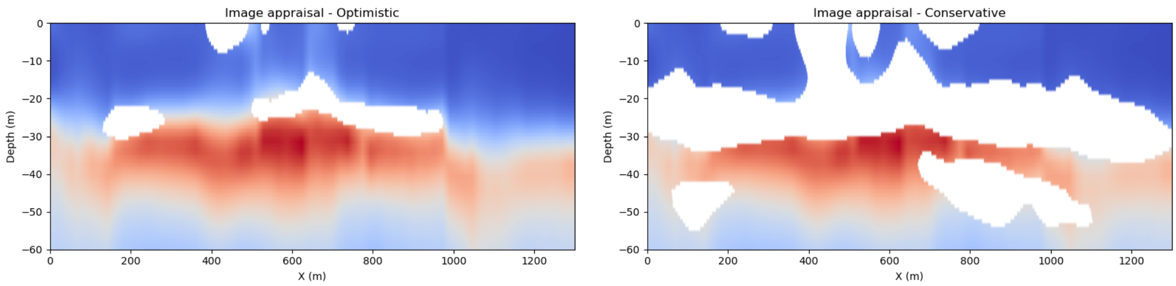

Figure 2 *Optimistic and conservative image appraisal of quasi 2D inversion models. White zones should not be interpreted quantitatively.*

Conclusions

We have proposed a computationally inexpensive image appraisal tool for AEM inversion. It enables to assess an inversion model obtained with a low fidelity (approximate) forward model for areas that are not fitting the true multidimensionality of the observed data. Adding this step to any quasi 2D or 3D method, which only involves one full 2.5D or 3D forward and one reduced Jacobian computation, prevents from quantitatively interpreting problematic areas in the inversion model.

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